

1 Introduction

Welcome to Pleasanton Math Circle (PMC)! Today we will be playing some math and logic games, and we hope you have fun!

2 Warm Up

Congrats! You are on a game show in which you strike a deal: you can either get 5000 dollars guaranteed or get one million dollars with only a .0001% chance of actually getting the money. Which do you choose? Explain your reasoning.

- What if there was a .001% chance?
- What if there was a .01% chance?
- What is the lowest percent chance of winning the million dollars at which it would be worth the risk? Why?

3 The Locker Room

There is a room with a certain number of lockers that are all open. You decide to play a game where you close all the lockers. The next round you open every other locker until you reach then end. After that you start at the beginning changing the status of every third locker. You continue this until you go through as many rounds as there are lockers in the room. After you finish the game, how many lockers are closed if there are

1. 50 lockers?
2. 1000 lockers?
3. What is the pattern?

4 Truth Or Lie

Tweedledum and Tweedledee, who are siblings, look alike, but Tweedledum lies on Monday, Tuesday, and Wednesday, whereas Tweedledee lies on Thursday, Friday, and Saturday. They both tell the truth on Sunday. You come upon the two of them, and they make the following statements. In each case you don't know who person A or B is, determine who is who, and what day it is.

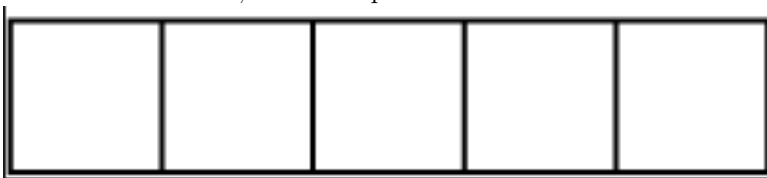
- A: I will lie tomorrow. B: I lied yesterday, and I will lie tomorrow.
Day and Person:
- A: My brother tells the truth. B: Today is Saturday.
Day and Person:
- A: I am Tweedledum. B: I am Tweedledee.
Day and Person:

5 Pirate Puzzle

There are five pirates trying to split a treasure of 1000 gold coins. They decide to split the coins using this scheme: The captain proposes how to share the coins, and all on board pirates (including the the one who proposed the plan) vote for or against it. If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the next highest in command proposing the plan. As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard. Assuming that all 5 pirates are perfect logicians, what will happen?

6 Catch the Leprechaun

A week before St. Patrick's Day, a sly leprechaun is hiding from you, trying to protect his pot of gold. There are five bushes in a row, and the leprechaun hides in one of these boxes.



Each night, the leprechaun moves one bush to the left or right, hiding in an adjacent bush the next day. Every morning, you can look in one bush to try to find the leprechaun. How can you guarantee you find the pot of gold before St. Patrick's day?

7 Magic Duels

You and 2 other wizards (you, the mediocre magician, and the victorious enchantress) are having a duel. You guys stand in a triangular shape. The rules are as follow: Only one person can cast a spell at a time and they can cast to anyone they want however the order in which the wizards take turns is you first then the Magician and then the Enchantress. The Magician has a 70 percent chance of successfully spelling someone while the Enchantress has a 90 percent chance. If someone is spelled they are out. However if no one gets out by the end each round then everyone loses! Now you get to pick a wand(wand options shown below). If you want to have the BEST CHANCE of winning which wand do you pick.(Remember your opponents are very smart!)

- The Bronze wand: Spells perfectly 60 percent of the time
- The Silver wand: Spells perfectly 80 percent of the time
- The Gold wand: Spells perfectly 100 percent of the time

8 Challenge

Bob and Alice are captured by the mean logician. In their cells Alice can see 12 trees and Bob can see 8. They are told that they can see all the trees but no tree is seen by both people. Everyday Alice is asked by the logician, "Are there 18 or 20 trees in total?" She has the option to answer or pass. If she chooses to pass the question is then passed to Bob and if he chooses to pass then the same happens the next day. If one person answers incorrectly then the two are imprisoned forever. However if they answer correctly they are free. Bob and Alice cannot communicate with each other however they know that each of them are asked the same question and they also know when one of the other passed. How can the two answer the question right with certainty(without guessing)?