1 Introduction

Welcome to Pleasanton Math Circle (PMC)! Today we will be working with sequences, but what is a sequence? A sequence is any objects(in this case numbers) that follow a certain pattern such as 10, 20, 30, and so on.

2 Fractals

Fractals are complex patterns that are similar across different scales. To make them you repeat a similar process in a never-ending loop.

- Circles: Starting at the edge of your paper, try drawing a circle that takes up half your page. Next to it draw a circle of half its diameter and continue drawing circles of decreasing diameters until you can't make them any smaller.

  1. How many circles were you able to draw?
  2. Is it possible to reach the end of that page? Why or why not?
  3. Given this, what would you get if you added 1/2 + 1/4 + 1/8 + 1/16... and so on?

- Sierpinski's Triangle: Draw an equilateral triangle and mark the midpoint of each of the sides. Now, create a new triangle connecting the dots forming three smaller triangles in each corner. Repeat the process for the smaller triangles, but remember you don't need to fill in triangles you draw or in order words all the new triangles should face the opposite direction of the main one.

- Now try creating your own. Remember the properties a fractal must have.

3 The Fibonacci Sequence

Now let's try a world famous sequence, the Fibonacci sequence. No matter how famous it is, it is quite simple in the sense that it starts off with two ones. Take 1 + 1 (write 1 and 1 in the first 2 boxes below and its sum in the third). Now take that sum and add it to the previous number(in this case 1) and write that sum number in the fourth box. Now take that number and add it to the previous sum (in this case the number in the third box). Continue this pattern until the table is filled out. Now that we have part of the sequence lets try to find out why it is so special.

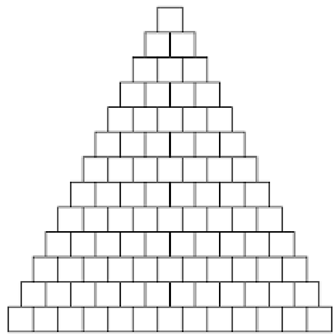
↓ ↓ ↓ ↓ ↓

1. Look at every 3rd number, what do you notice?
2. Take every 2 consecutive number starting with 1 and 1 and divide them (ex: 1/1) and so on (you may need a calculator) what do you notice?
3. Now take the first 5 numbers in the sequence and multiply each number by itself. Write your answer under the arrow. Then add your products to the products next to it one by one. What do you notice?
4. Divide each original Fibonacci number by 2 and write down your remainder. What do you notice? What do you think will happen if you did the same thing but instead of dividing by 2 you divided by 3 or 4? Hint: The remainder of 1/2 is 1 and the remainder of 2/2 is 0.

4 Pascal's Triangle

Pascal's Triangle is a number pattern in the shape of a triangle named after French mathematician Blaise Pascal. Here is how to build Pascal's Triangle. Given below is an empty pyramid of boxes.

1. Fill in the top box with the number 1.
2. Fill the second line in with two 1's. For every line after, fill in the boxes on the sides with 1's.
3. Fill in each interior box with the sum of the values in the two boxes directly on top. For example, the third row will be {1, 2, 1} and the fourth row will be {1, 3, 3, 1}.



1. After filling out the triangle, what patterns in the numbers do you notice?
2. What if you were to take the sum of each row, what pattern do you see?
3. Now, color or shade in the boxes with odd values, do see anything familiar?
4. Challenge: Find a way to get the numbers of the Fibonacci sequence from Pascal's Triangle.

Pascal's Triangle can be used to find the number of combinations. The number of combinations are the number of ways to choose subgroups from a group of items. For example, say you have 5 colored marbles in a bag {red, blue, green, yellow, orange}. The different combinations of two marbles are {red/blue, red/green, red/yellow, red/orange, blue/green, blue/yellow, blue/orange, green/yellow, green/orange, yellow/orange}. There are 10 combinations of two colored marbles from the bag. On Pascal's triangle, the way you can find the number of combinations for 2 marbles from 5 marbles is the 3rd entry of the 6th row.

1. Let's say you want to choose 3 students from a group of 4 students {Al, Bob, Carl, Dan}. What are the possible combinations of 3 students? Where do you find the number of combinations on Pascal's Triangle?
2. Now, say you want to choose 4 officers from a committee of 8 people? Without listing out the combinations, can you find the number of combinations from Pascal's Triangle?

5 Challenge

In Greek mythology, a creature known as the hydra is believed to sprout two new heads in place of one that has been cut off. Recreate this by starting at the top of your page with a wide upside-down V. from each endpoint, draw another downwards V and continue this pattern.

- Now we will add in some numbers. Try this puzzle: At the top of your page, write 1/2 but expanded to 1/(1+1). Now each of the 1s in the denominator can be replaced with 1/ (1+1) approaching infinity. Remember 1 = 1/1. What do you think it will equal?
- Now try solving the first 5 rows. What did you get? What can you conclude about this puzzle?
- This time, lets work from the bottom up. Starting with (1+1)/1 at the bottom of your paper and replace the 1s in the numerator with (1+1)/1. What do you think it will equal as it approaches infinity?
- Again, solve the first 5 rows. What do you notice?
- Bonus: if you replace the two 1s directly above the numerator with x, what happens when you solve the equation setting it equal to x?