

1 Introduction

When solving problems, you often need to know more than just math; you need to also know logic. Logic is a way of thinking about the reasoning behind a question. Sometimes, to solve a difficult question, you don't have to know any fancy math—all you have to do is think about the question logically, and the answer will be apparent. Other times, you need to use logic to figure out how to solve a question and math to actually solve it. We will be doing all both types of logic problems today. Enjoy!

2 Warm-up: Fruit in Jars

You have three jars that are all mislabeled. One contains apples, another has grapes, and the third has a mix of both.

Now you can open any one jar to see one fruit. [The jar you open may contain one or two fruits, but you are only able to see one fruit, and you can't see whether the opened jar has one or two fruit]. How can you label the jars correctly?

3 Knights and Liars

There is an island far away, whose inhabitants are quite unusual. Some of them never tell a lie, and others never utter a true statement. Those that who never lie call themselves Knights. Those who never tell the truth are known as Liars. Even though they have different personalities, knights and liars cannot be told apart by appearance: they wear the same types of clothes, and have the same haircuts. Sometimes tourists visit this island. The Tourists are regular people: sometimes they lie, and sometimes they tell the truth. A few of the following problems are about the inhabitants of this island.

1. While visiting this island, you meet a boy who says he a liar. Does he live on this island?
2. Two island boys, Sam and Kurt, are introducing themselves to you. Sam says, "At least one of us is a liar." Who is/are the liar(s)?
3. While visiting this island, I had a conversation with a local knight. I asked him the same question twice, and he gave me two different answers. What was my question?
4. At the island, you meet two islanders, Ben and Jerry. Ben says at least one of the two is a liar. Is Ben a knight or a liar? What about Jerry?
5. While visiting the island, you pass a beautiful garden where you meet three islanders, Kevin, James, and Kyle. You ask Kevin, "Are you a Knight or a Liar?" Kevin ignores you. You then ask James, "Is Kevin a Knight or a Liar?" James says, "Kevin is a liar." Then, Kyle screams, "Don't listen to James! He's a liar!" Is James a liar? Is Kyle a liar?

4 A Knight and a Dragon

Let's consider a dragon and knight who are living on an island. The island has seven poisoned wells, which are numbered 1 to 7. The wells are very dangerous because the poison looks like regular water. If you drink one glass of poison from a well, you can only save yourself by drinking another glass of poison but from a higher numbered well. For example, if you drink poison from well 2, you can drink poison from wells 3-7, and you will live. You cannot drink poison from wells 1 or 2, or else you will die. The highest number well is well 7, but it is at the tippity top of a high mountain so only the dragon can reach it.

One day the knight and dragon decide that the island isn't big enough for the two of them, and they have a duel. Both the dragon and the knight get a glass of poison and their opponent has to drink it. Afterwards, they can drink poison from any of the wells. The next morning, the knight is alive and the dragon is dead.

Why did the knight live? Why did the dragon die?

5 Bad Children and Halloween Candy

Every Mom instantly knows when a child other than her own has misbehaved, but does not know when her own child has. There is an unspoken rule between Moms that any child who misbehaves will have their Halloween candy taken away. Any Mom who can find out that their child misbehaves will take away the candy. One day, the principal announces that at least one child has misbehaved.

1. What happens when there is one misbehaving child?
2. How about two?
3. How many days does it take for punishment to occur if there are 100 misbehaving children?

6 Challenge: Logic and Math

A bad king has a cellar of 1000 bottles of very tasty soda. A neighboring queen plots to kill the bad king and sends a servant to poison the soda. Fortunately (or say unfortunately) the bad king's guards catch the servant after he has only poisoned one bottle. The guards don't know which bottle but know that the poison is very strong. Also, it takes one month to have an effect. The bad king decides he will get some of the prisoners in his vast dungeons to drink the soda. Being a clever bad king he knows he needs only ten prisoners to drink the bottles of soda to figure out which one is poisoned, and he will be able to drink the rest of the soda on his birthday in 2 months. Explain how the king will be able to only use 10 prisoners to figure out which bottle is poisoned.