

## 1 Warm Up

Solve for the sum.

1.  $E_1 : 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \text{-----}$
2.  $E_2 : 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 \dots = \text{-----}$
3.  $E_3 : 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 \dots = \text{-----}$
4.  $E_4 : 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 \dots = \text{-----}$

## 2 The Infinity Inn

You own the Infinity Inn with an infinite amount of rooms. For each person who stays at your inn, you get an infinite amount of money, so it is in your best interest to make space for everyone. At the moment every room is full. The maximum capacity of each room is only one person.

With these requirements how would you accommodate...

1. 1 person?
2. 100 people?
3. Infinity people?

Challenge: You've gotten famous! An infinite amount of tour buses come each filled with an infinite amount of people. How do you lodge all of them?

Bonus: Are you making more money by accommodating an infinite amount of people than by accommodating 1 person? Remember each person gives you an infinite amount of money.

## 3 How Big is Infinity

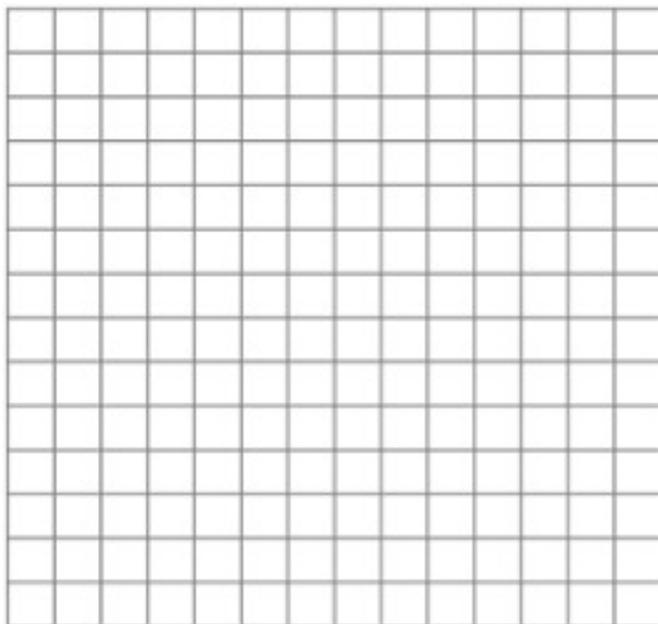
Infinity is defined as having **no limit**. There are different types of infinity, the lowest level being countable infinity of natural numbers. **Natural numbers** are the counting numbers  $1, 2, 3, 4, \dots$ , so it doesn't contain fractions, negatives, or irrational numbers.

Predict: Are there more even numbers or whole numbers?

For this activity, think of comparing numbers as matching up the items from each list. Because you don't need to know the specific amount, this method allows you to compare without counting.

You loaned out five books to your friend, how can you confirm that all were returned?

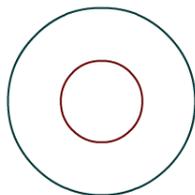
1. Now use this method to verify your prediction. Are there more even numbers than whole numbers? Explain.
2. Try writing out all the fractions on the table on the next page. Can you do it?



3. Is it possible to write out all the decimals?

- How many one-digit numbers are there?
- How many different ways can you make a unique 2 digit number?
- How many ways can you make a unique decimal (less than one) with three digits after the decimal?
- How many ways can you make a unique decimal with an infinite number of digits?

So can you write out all of the digits?



## 4 Set Theory

- Make a set of numbers.
- Now, create a new set using subsets of your original set.
- Is your set of all the subsets bigger than your original set? Explain.

You can create bigger infinities by making a set of all subsets of an original set and then creating another one of all the subset of the new set and so on. So there is an infinite number of infinities of different sizes. The **Continuum Hypothesis** states that there are different sizes of infinities, and it is the most important unsolved mysteries of math.