1 Warm Up

Solve for the sum.

1. **55** add the first and last number to get 11. Since there are 5 total pairs, multiply by 5, resulting in 55.
2. **1/2** just looking at the first two terms results in 0. Looking at the first three terms results in 1. Since we don't know whether the last term is +1 or -1, take the average of the two answers ((0+1)/2) to get 1/2. This is known as **Grandi's series**... there are other ways to prove it as well.
3. **1/4** first, add the equation to itself:
   1-2+3-4+5-6+7...
   + 1-2+3-4+5-6...
this can be simplified into 1-1+1-1+1... which is already proved as 1/2, but since this is double the series, divide 1/2 by 2 to get 1/4.
4. **-1/12** to solve this, subtract the previous series (E3) from it.
   1+2+3+4+5+6+7+8...
   -(1-2+3-4+5-6+7-8...)
note that by distributing the negative causes the signs to change. By computing this, the answer is 4+8+12+16+20... which are all multiples of 4. So, it the answer can be written as 4(1+2+3+4...). Set up the final equation E4 - E3 = 4(E4). Since it is proved that E3 is 1/4, E4 - 1/4 = 4(E4). Isolate the variable and solve to get E4=-1/12.

2 The Infinity Inn

How would you accommodate...

1. 1 person? *Move everyone to one room up. n to n+1 and let the new person take room 1.*
2. 100 people? *Move everyone up 100 rooms. n to n+100 and let them fill the first hundred rooms.*
3. Infinity people? *Move everyone to the room of twice their room number. n to 2n. We already proved that there are an infinite number of even and odd numbers, so the new infinity people can take the odd numbered rooms and the current members can stay in the even ones.*

Challenge: *There is an infinite amount of prime numbers, proved by Euclid. Take the first prime number, 2, and move each of the existing residents to 2 to the power of their room number. So someone in room n would move to 2 to the power of n. So a person in room 5 would go to Room 32. Now, for the first bus, go to the next prime number, 3, and have each person go to 3 to the power of their seat number.So someone on the first bus with seat n would move to the room number that is 3 to the power of n. Assign each of the buses to the next prime number and then have the passengers occupy to room of that number raised to their seat number. So it would be x raised to the nth power, as in prime number and n is seat number. There will be extra rooms for numbers like 6 (2x3) that aren't powers of whole numbers, but now you can seat more guests!*

Bonus: Are you making more money by accommodating an infinite amount of people than by accommodating 1 person? Remember each person gives you an infinite amount of money.

3 How Big is Infinity

You loaned out five books to your friend, how can you confirm that all were returned?
Make a tally for each five books and as each one is returned erase a mark. As long as there is no marks left over, the two lists are equal.

1. Now, answer the initial question. Are there more even numbers than whole numbers? Explain. *write out 10 the whole numbers. Now below that write out double of the number. n to 2n.*

1/1	2/1	3/1	4/1	5/1	
1/2	2/2	3/2	4/2	5/2	
1/3	2/3	3/3	4/3	5/3	
1/4	2/4	3/4	4/4	5/4	
1/5	2/5	3/5	4/5	5/5	

2. Try writing out all the fractions. Can you do it?
3. Is it possible to write out all the decimals?
   (a) How many one-digit numbers are there? *10. 0-9*
   (b) How many different ways can you make a unique 2 digit number? *90. 9x10 9 possibilities for the tens digit and 10 for the ones.*
   (c) How many ways can you make a unique decimal (less than one) with three digits after the decimal? *900. 9x10x10. 10 possibilities for tenths, 10 for hundredths, and 9 for thousandths.*
   (d) How many ways can you make a unique decimal with an infinite number of digits? *Infinite. 10 to the power of infinity*

So can you write out all of the digits? No. For any decimal change one digit and it is different.

4 Set Theory

1. Make a set of numbers. *ex. whole numbers*
2. Now, create a new set using subsets of your original set. *ex. 1,2,3, 1,2, 24,36,48*
3. Is your set of all the subsets bigger than your original set? Explain. *Yes*

You can create bigger infinities by making a set of all subsets of an original set and then creating another one of all the subset of the new set and so on. So there is an infinite number of infinities of different sizes. The **Continuum Hypothesis** states that there are different sizes of infinities, and it is the most important unsolved mysteries of math.