

1 Number Counting

How many two-digit numbers are not divisible by 3? Instead of counting 10, 11, 13, 14 and so on, let's try a new technique, called complementary counting.

1. How many two-digits numbers are there?

90

2. How many two digit numbers are divisible by 3? (Hint: Find the smallest and largest two-digit number divisible by 3)

30

3. So, how many two-digit numbers are NOT divisible by 3?

60

Based on this example, what is complementary counting? Instead of counting the numbers that we do want, we did something much easier- the exact opposite.

Here are the steps we took:

1. First, we counted the total possibilities (all two-digit numbers).
2. Next, we counted the number of possibilities that don't work (two-digit numbers that are divisible by 3).
3. Finally, by subtracting the number of possibilities that don't work from the total number of possibilities, we found the number of possibilities that do work (two-digit numbers that are not divisible by 3).

To summarize: Possibilities that DO work = TOTAL possibilities - Possibilities that DON'T WORK

2 More Number Counting

Let's find the number of three-digit numbers not divisible by 5 (Use steps 1, 2, and 3 to solve this!)

1. Step 1 (total):

900

2. Step 2 (do not work):

180

3. Step 3 (answer- do work):

720

3 Dinner Tables

The Thomas family has 2 daughters and 3 sons. For dinner, all the kids sit on one side of the table in a row. In how many ways can the seating be arranged such that the 2 girls are not sitting next to each other? (The girls like to gossip too much, so the parents want to make sure they actually eat!)

1. Count the TOTAL number of possible seating arrangements.

10

2. How many ways can we arrange the seats so that the two girls ARE sitting next to each other?

4

3. So, how many ways can we arrange the seats such that the two girls ARE NOT sitting next to each other?

6

4 More Problems

Use the complementary counting to solve these problems!

1. How many words can we make out of the letters H, I, A, O, and P such that each word has at least one consonant?

310

2. Anjaber is looking through her closet, and she sees three shirts (red, white, black), two pants (black, blue), and three shoes (brown, white, black). How many ways can Anjaber pick an outfit with no matching colors (for instance, she can not choose a white shirt, black pants, and white shoes, since both her shirt AND her shoes are white)?

10

3. How many 3 digit numbers don't have a 4? (for instance, 312 fits our description, but 924 does not)

648

4. I went shopping and got a bunch of new clothes. I have 8 different hats, shirts, pants, and shoes, all of the same 8 colors (red, orange, yellow, green, blue, purple, black, white). How many ways can I put together an outfit without having my hat, shirt, pants, or shoes be the same color?

31

5. I'm eating dinner with my 9 other friends, but two of them, Parthemey and Shawkar, refuse to sit next to each other. And, we're sitting in a row of 9 seats. How many ways can I create a seating chart that will make everyone happy?

282240

6. There are 30 students at Pleasanton Math Circle. What is the probability that at least two students have the same birthday?

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