

1 Fake Coins

Antique Collector Amber was going through her old coin collections when she found out that she had many real coins, as well as many counterfeit ones. She decided to experiment with them in various ways, but then she realized that she couldn't tell the difference between the real and fake coins! Help her distinguish between them. In the problems below, each real coin weighs 10 grams, but each fake coin weighs 9 grams.

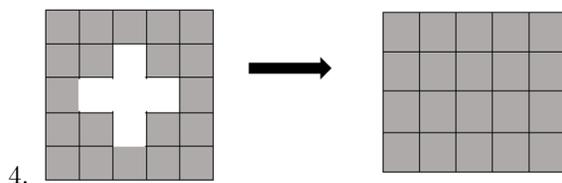
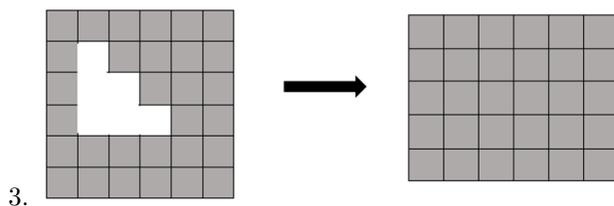
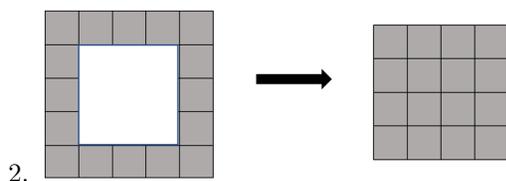
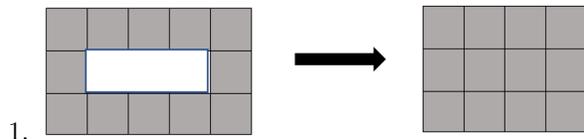
1. You have two bags full of coins, one filled with real coins and the other filled with fake coins. Suppose you put the coins from only one of the bags on a digital scale. Will you be able to figure out which bag contains fake coins?
2. Now, you put one coin from each of the bags on the scale. Will you be able to determine which bag contains fake coins? Can you think of your own method to figure out which bag has the fake coins?
3. Now suppose you have 3 bags full of coins. Again, one of the bags holds fake coins. How can you determine which bag has fake coins in just one weighing? How about 10 bags; how can you figure it out in only one weighing?
4. Based on your findings, create a small table detailing how you would be able to figure out which bag has fake coins with larger numbers of bags. For example, suppose there were 20 bags. How would you figure out which one contained the fake?

For the following problem, imagine that you are using a two-pan balance scale.

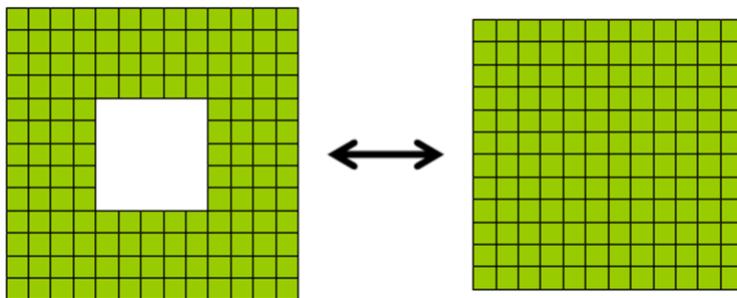
5. You have 9 coins and 1 of them is fake. The real coins weigh more than the fake coins. Using a balance scale, what is the minimum number of weighings that you would need to figure out which coin is the fake coin?

2 Quilt Mending

Prince Parth has damaged all of his quilts during sword practice, and now they all have holes in them. Prince Parth needs help fixing the quilts so that they look like full quilts once again. For each of the following, show how to cut the quilt on the left to create the quilt on the right using the least amount of PIECES possible.



5. CHALLENGE: Jester Jeremy claims that he can mend the following quilt with only 5 pieces, but Prince Parth doesn't believe him and says it requires more pieces. Who is right?



6. BONUS: Artist Anjali is ashamed that all of Prince Parth's quilts are gray, and wants to add some color to them. However, her rule is that two squares that share a common edge or even touching corners cannot be colored the same color. Find the fewest amount of colors that should be used to paint the finished quilt in question 5.