

## 1 Warm Up

Before we look at the specific pattern in the Fibonacci Sequence, let's warm up with some patterns. In each set of numbers below, identify the pattern and fill in the blanks. Keep in mind that the patterns might use things besides addition, subtraction, multiplication, and division.

- 1, 3, 5, 7, 9, 11, 13
- 1, 2, 4, 8, 16, 32, 64, 128, 256
- 1, 4, 9, 16, 25, 36, 49, 64, 81
- 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
- 1, 3, 6, 10, 15, 21, 28, 36
- 18, 12, 19, 11, 20, 10, 21, 9

## 2 The Fibonacci Sequence

Now that we have closely examined some mathematical patterns, let's take a look at one of the most famous sequences of numbers in mathematics: the Fibonacci Sequence.

- The first few terms in the Fibonacci sequence are 1, 1, 2, 3, 5, 8, 13. What is the pattern?

*Each number is the sum of the two previous numbers.*

- Find the next 9 terms in the sequence, and write them below.

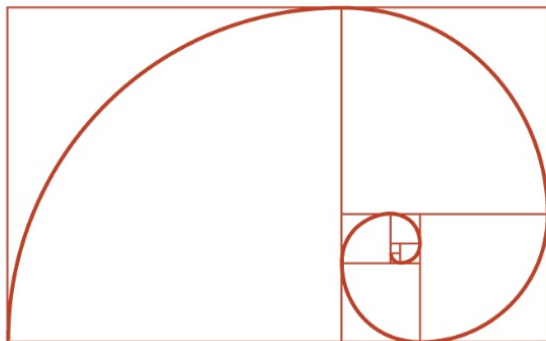
*21, 34, 55, 89, 144, 233, 377, 610, 987*

- You should now have the first 16 terms of the Fibonacci Sequence written. Now, try dividing the 2nd term by the 1st, the 3rd by the 2nd, the 4th by the 3rd, and so on. Can you find a pattern of any kind?

*The ratio should near the golden ratio: 1.618.*

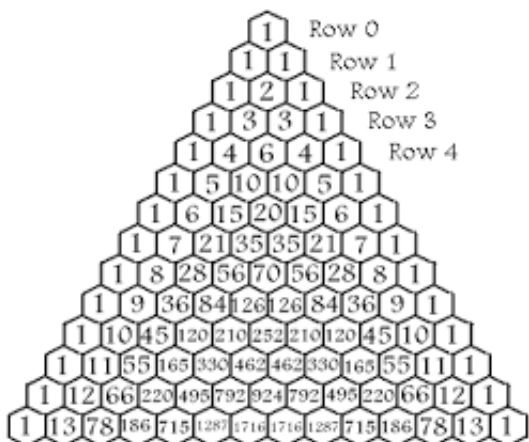
## 3 The Golden Spiral

The number you just found is known as the golden ratio, an important value for artists and architects. Now let's take a look at why this ratio is so fascinating, using the Fibonacci Sequence. Color the grid below using the Fibonacci Sequence as demonstrated on the board.



## 4 Pascal's Triangle

One of the most interesting number patterns is Pascal's Triangle (named after Blaise Pascal, a famous French Mathematician and Philosopher). To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern. Each number is the sum of the two numbers directly above it. Try filling out Pascal's Triangle below.



1. How is this related to the Fibonacci Sequence? Where can you find the Fibonacci Sequence hidden in Pascal's Triangle?

*The sum of the diagonals gives the Fibonacci sequence.*

2. Now, try to find some patterns in Pascal's Triangle. There are lots to see, so try finding as many as you can!

*There are lots of patterns in Pascal's Triangle, but here are some of the most common ones: hockey-stick-shaped sum rule, the numbers in each row form a power of 11, the sum of the numbers in each row are powers of 2, the first diagonal on either side consists of ones, the second diagonal on either side consists of counting numbers, the third diagonal on either side consists of the triangular numbers, etc.*

## 5 Tribonacci

1. Try to construct a sequence using the pattern of the Fibonacci sequence, but instead having 1 and 3 as the first two numbers.

1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199

2. Now, instead of adding the 2 previous numbers together to get the next number in the sequence, let's try adding the 3 previous numbers. Find 6 more terms in the sequence below.

1, 1, 2... 4, 7, 13, 24, 44, 81

3. CHALLENGE: Can you also find the ratio (fraction) between each term and the previous one for this sequence?

*It is around 1.84.*