

1 What is a Sequence?

What are mathematical sequences? A sequence, in mathematics, is a string of objects, like numbers, that follow a particular pattern. For instance, the series of numbers 1, 10, 100, 1000... is a sequence because it follows a very specific pattern.

2 Identify the Sequence

Take a look at these sequences and try to figure out the pattern behind them:

- 1, -1, 1, -1, 1, -1...
- 4, 8, 12, 16, 20...
- 2, 5, 8, 11, 14...
- 1, 3, 6, 10, 15, 21...
- 1, 2, 6, 24, 120...
- 1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211...
- 1, 4, 16, 28, 40, 16, 28, 40, 16, 28...

3 Visual Series

3.1 Square Series

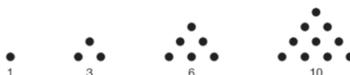
Draw a one by one square represented by a single dot. Now, draw a 2x2 square represented by dots (Hint: there should be a total of 4 dots!) IT should look something like this:



Continue drawing these dotted squares until you have drawn a 6x6 square. What do you notice about the total number of dots? What kind of series is this? Can we create an equation to find the number of dots?

3.2 Triangle Series

Create a series of triangular numbers. Take a look at the picture below and continue the sequence. Can we



find an equation for the total number of dots in the triangle series?

4 Fibonacci

We're going to move on to more difficult sequences. Let's start with two numbers: 1 and 1. Add the two numbers together. What number should that give you?

Next, add the last number you just found to the number right before. Keep doing that until you have a total of 12 terms:

Now, try dividing one term by the term before. You should start out with $\frac{1}{1}$. Keep continuing. You may need a calculator for this. Do you notice a pattern?

That's just one cool property of the Fibonacci sequence. Here are a couple others (try to verify them!):

1. The sum of any 10 consecutive Fibonacci numbers is divisible by 11
2. Two consecutive Fibonacci numbers do not have any common factor, which means that they are Co-prime or relatively prime to each other.
3. The Fibonacci numbers in the composite-number (i.e., non-prime) positions are also composite numbers.

5 Fibonacci 2

Last time, we started with the numbers 1 and 1, but what happens if we follow the same Fibonacci rule starting with the number 2 and 1?

Fill in the rest:
2, 1, 3, 4...

This is called a Lucas Series. Check to see if some of the properties above hold true. Does the ratio between one number to the next still get closer to the Golden Ratio?

Next, write out the Fibonacci numbers next to the Lucas numbers. It should look something like this:

n:	0	1	2	3	4
F_n:	0	1	1	2	3
L_n:	2	1	3	4	7

Do you notice any similarities between the Fibonacci numbers and the Lucas numbers? (HINT: add two of the numbers to get another number)

What happens when we add the numbers two spaces away from the Lucas numbers?