

1 The Monty Hall Problem

1. In the beginning, what is the probability of picking the door with a car behind it? (This is the probability of eventually getting a car if you do not switch.)

1/3 (one of the three doors holds a car behind it.)

2. In the beginning, what is the probability of picking the door with a goat behind it? (This is the probability of eventually getting a car if you do switch.)

2/3 (two of the three doors hold a goat behind it.)

3. After the game show host opens the door with a goat behind it, what is the probability of having a goat behind your door? A car?

The probability of having the goat is still 2/3, while the probability of having the car is still 1/3, as in the previous answers above.

4. Does the the probability of you picking either a car or a goat on the first try change if the game show host reveals one of the unpicked doors?

No, both probabilities remain unchanged.

5. What is the probability that the unopened door you did not pick contains a goat? A car?

The probability that the unopened door holds the goat is 1/3, while the probability it has the car is 2/3.

6. Now that we've gone over the logic, what should you do?

You should switch doors in order to increase your chances of winning the car.

7. In order to double check our logic above, fill out the table below to see by how much switching doors increases our probability of winning the car. Fill in each blank space with either STAY or SWITCH based on what you should do in the given situations.

	Car Is Behind Door #1	Car Is Behind Door #2	Car Is Behind Door #3
You Pick Door #1	Stay	Switch	Switch
You Pick Door #1	Switch	Stay	Switch
You Pick Door #1	Switch	Switch	Stay

8. Using the information you filled out in the table above, by how much more likely is it to win the car if you switch than if you stay?

It is twice as likely to win the car if you switch. There is a 2/3 chance if you switch, but only a 1/3 chance if you stay with your original door of choice.

2 Monty Hall Bonus Round

1. In the beginning, what is the probability of picking the door with a car behind it? A goat?

The probability of picking a car is $1/100$, and picking a goat has a probability of $99/100$.

2. Now, after the game show host opens the door with a goat behind it, what is the probability of having a goat behind your door? A car?

Both probabilities remain the same ($1/100$ of having the car and $99/100$ for having a goat).

3. What is the probability that the unopened door you did not pick contains a goat? A car?

The probability that the unopened door holds the goat is $1/100$, while the probability it has the car is $99/100$.

4. Therefore, what should you do?

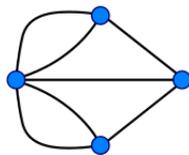
As with the first situation, you should switch doors.

3 The Seven Bridges of Königsberg

1. Does it matter what shape the rivers are in? What about the bridges?

No, it does not matter what shape the rivers or bridges are. Only the number of bridges and where they connect affects the problem.

2. Now, redraw the picture below, replacing land with *vertices* or dots and replacing rivers with *edges* or lines (they can be curved or straight).



3. What do you notice about the number of lines attached to each vertex?

The number of lines attached to each vertex is ODD in every case.

4. Now, if every bridge can only be crossed exactly once, what must be true about the number of bridges touching a certain land mass (or vertex)?

In order for the conditions to be met, there must be an EVEN number of bridges touching each land mass.

5. Therefore, what can you conclude about this seemingly solvable problem?

The problem is unsolvable given the layout in the picture above.