1 Warm Up

Here is a great warm-up game. There is one pile of 21 counters. On your turn, you can take 1, 2, or 3 counters. You win if you take the last counter. With a partner, take turns crossing off 1, 2, or 3 counters and see if you can end up with the last one.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

Let's change the rules up a bit. Now, you can still take only 1, 2, or 3 counters, but the person who takes the last counter loses. Can you figure out a new strategy to win?

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

2 Nim

We're going to play a game called Nim. First, draw three circles with a random number of squares in each pile. For example, you could draw 2, 3, and 6 squares in each of the three piles. Then, taking turns with your partner, on your turn, pick one pile and remove one or more squares from the pile. Keep alternating turns until there are no more squares left. To win, you must NOT be the person to remove the last square from the entire board. In other words, make sure your opponent crosses off the last square.

3 Nim Challenges

For the first three questions, assume that you are the first person to go.

1. Suppose you have two piles of two squares each (2 + 2). Assuming your opponent makes the smartest move possible, is there any move you can make which will guarantee that you always win?

   No

2. How about with 3 + 2?

   Yes, take one from the pile of three, which makes it a 2 + 2 situation, but it's your opponent's turn now.

   Situations in which you will always lose, no matter what you do, are called losing positions. However, if there is a strategic move that will guarantee a win for you, then you are in a winning position.

3. Between questions 1 and 2, which one is the losing position and which one is the winning position? Why? In the winning position, what is the strategic move you can make?

   1 is the losing position, and 2 is the winning position. The move is described above.

   For the next two questions, assume that your opponent is the first person to go.

4. Can you think of a situation with two piles where you will always win, no matter what your opponent does? In other words, can you think of a situation where your opponent always leaves you in a winning position?

   3 + 3

5. Now can you think of a situation where you will always lose (losing position), assuming your opponent makes the smartest move possible?

   Trick question, can be anything that is not a winning position, because the opponent will make the smartest move possible, so if there is a situation in which you will win in all but one move, like 4 + 3, you will still lose because your opponent will make that one move.

4 The Fabulous Game of Jim (Jeremy's Nim)

Jeremy was playing Nim when he got an idea: he decided to make his own version of the game!

To begin a game of Jim, you start with several rows of solid tokens and empty tokens. Players alternate moves: you select a row and change one or more tokens (solid to empty or empty to solid). The first token-change from the left must be solid to empty (but it does not need to be the leftmost solid token). The last player with a legal move wins. That means, if you see only empty tokens, you lose!

1. Here's an example of a simple game of Jim. Suppose you start with one row, with this arrangement. What can you do?

   You can't do anything, you have already lost.

2. Now try this arrangement. Assume that you go first.

   Just change both the solids to empty.

3. Which of the previous two are "winning" and "losing" arrangements?

   1 is a losing position while 2 is a winning position.

4. How about two row Jim? (The line is separating the two rows.) Which of the following are losing and winning positions?

   First is losing position, second is winning position

5 Nim and Jim

1. Is there any way to return any Jim sequence to its original sequence of empty and solid tokens? For time purposes, try the arrangement in question 2 of the previous section, with only one row.

   No

2. What is the maximum number of legal moves that will change the first sequence to the second sequence?

   6

3. What one-pile Nim game has the same maximum number of legal moves?

   A pile with as many counters as moves i.e. if it was 2 moves, then 2 counters; in this case 6 counters