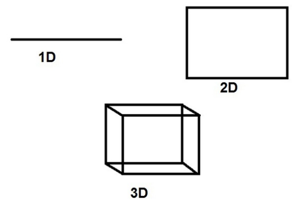
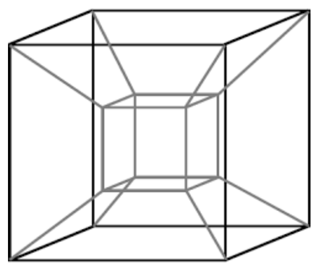
1 What is a Dimension? 1D to 4D



How would you define a dimension? We know 0D is a point, 1D is a line, 2D can be represented in shapes like squares and circles, and 3D can be represented in spheres and cubes, but what defines these dimensions?

Dimensions can be related to variables. One dimension is one variable-for example, the dimension of a line is its length or its x-value- two dimensions has two variables (x and y values), and three dimensions relates to three variables. Therefore, 4D should have 4 variables, but what is the 4th variable? Our space is described through 3 variables (x, y, and z) so what would 4 dimensions represent?



This is a physical representation of 4 dimensions, called a "hyper-cube" or a "tesseract". A common interpretation of the 4th dimension is time, therefore the 4 variables represented are x, y and z to represent space, and t to represent time.

2 How do you think 4D?

Take a plane - this will represent 2D. Now take a sphere (represents 3D objects) and move it across the plane. A plane "cuts" the sphere into circles of different size. Another way of thinking about it is by taking a lamp, holding a sphere in front of it, and casting a shadow on the wall. Because the wall is two dimensional, the shadow will be a two dimensional representation of the sphere. 4D and 3D objects have exactly the same relationship. A three dimensional object is just a cut or a shadow of a four dimensional object.

Practice

1. Consider the sequence D(n) = 5,6,3,3,.... What is the next term?
2. Can you make a 3D shape only using hexagons? Can you make a 4D shape using only hexagons? If yes, how?
3. These are 3D cross sections of a 4D cube passing through our 3D world edge first:

4. Draw 2D cross sections of a 3D cube passing through 2D world edge first. What do you notice? Why do you think it happens?

3 Graphs

Graphs show a relationship between things - if things are related, they are connected, if things are not related, they are not connected. We can take a group of people and connect all the people that know each other. This will be a graph of personal connections. We can also use graphs to understand how soccer players interact with each other by counting the number of times each player passed a ball to another particular player. From that, we can build strategies for our soccer team and make it a world champion!

Practice

1. Considering the fact that if a person A can know a person B, a person B does not have to know A, in how many ways can 3 people know each other? 4 people? 5 people?

2. What is the minimum number of colors needed to color in the map of Europe in a way that no bordering countries have the same color?

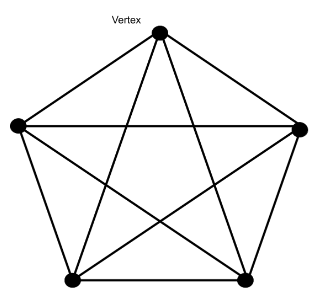
3. Draw a graph of a tetrahedron, a cube, and an octahedron.

4 You can draw all four dimensions?!

In order to be able to draw all four dimensions, we will use graphs, because they enable us to see all the connections between points happening at the same time from all the angles, while the three dimensional representation allows us to see only a piece of the whole 4D object.

In order to draw graphs of 4D shapes correctly, we need to understand how all points are connected, what faces those points create, and what happens to points once we scale the object one dimension. Let's take the simplest platonic solid - a tetrahedron. Notice that no matter what point we take on a tetrahedron, it will be connected to all its neighbours. It is also true for a triangle - a two dimensional "version" of a tetrahedron. In order to make a tetrahedron, we take a triangle and add a point above the triangle (in the new dimension) and connect all the points to that new point. This means that in order to make a hyper-tetrahedron, we need to add one more point and connect all the already existing points to the new one. As a result, we get this version of a hyper-tetrahedron:

But how do we turn that into a graph? Let's take a look at one of the points on the four dimensional tetrahedron - we know that it is connected to every other point on the graph. Thus, we draw five points and connect one of the points to the other four points. Every point on a tetrahedron has a property we found in the first point we picked, so we can connect each point to other 4 points in the same way. This is the resulting graph:



Practice

1. Draw a graph of a hyper-cube.
2. What would a 4D tetrahedron look like in a 3D projection? What will its graph look like?
3. Draw a 2D projection of a rotating 3D cube and then the 3D projection of a rotating 4D cube (only half of the rotation - the other half is the same).