

1 Introduction

Welcome to Pleasanton Math Circle (PMC)! Today we will be playing some math and logic games, and we hope you have fun!

2 Warm-Up

1. We are going to play a quick game involving some problem solving techniques. On your paper, write out the numbers 1 - 10, or use the numbers below. With a partner, take turns crossing off one or two numbers. If you remove the last number, you lose. Try it out a couple times, and see if you can come up with a strategy to always win the game.

1 2 3 4 5 6 7 8 9 10

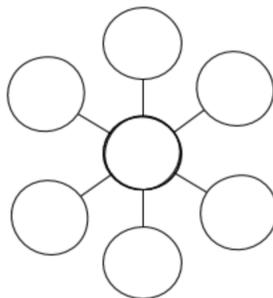
1 2 3 4 5 6 7 8 9 10

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2. Now, try the game again, except this time with 21 counters (the numbers 1 - 21). This time, you may cross off 1, 2, or 3 numbers on your turn. If you remove the last number, you lose. Try it out a couple times, and see if you can come up with a strategy to always win the game.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

3. Place the numbers 1, 2, 3, 4, 5, 6, and 7 in the seven circles drawn below so that any three numbers in a straight line have a sum of 14. Use each number once.



3 The PMC Hat Riddle

As part of the ancient PMC initiation ceremony, PMC lecturers tested their students' intelligence on the first day of class. They would randomly select a group of 4 PMC students and line them up from tallest to shortest. The lecturers then placed a black or white hat on each student. Starting with the person in the back, each person must say a single word: "black" or "white" to guess the color of his or her hat. However, the students had to face forward, so they couldn't look at their own hat. If three of the four students guessed correctly, they would each get a whole bag of candy. But if they didn't, no one would get any candy. Luckily, the lecturers allowed them to talk to each other beforehand so that they could formulate a strategy and pass the test. Can you and your table group figure out a way to pass the PMC initiation test? (There may or may not be candy involved.)

4 Nim

We're going to play a game called Nim. First, on your paper, draw three circles with a random number of squares in each pile. For example, you could draw 2, 3, and 6 squares in each of the three piles. Then, taking turns with your partner, on your turn, pick one pile and remove one or more squares from the pile. Keep alternating turns until there are no more squares left. To win, you must NOT be the person to remove the last square from the entire board. In other words, make sure your opponent crosses off the last square. As you play, try to come up with some strategies to win the game.

5 Crossing the River

There are four friends: Alpha, Beta, Gamma, and Delta, who have a boat and want to cross the River Styx. They only have one torch, and the river cannot be crossed without the torch. Also, there cannot be more than two people on the boat at any time, and when two people cross the river together, they must row at the slower person's pace. Alpha can row across in 1 minute, Beta in 2, Gamma in 5, and Delta in 8. But, they only have 15 minutes to cross the river before Hades notices and stops them. How can all four cross the river in time?

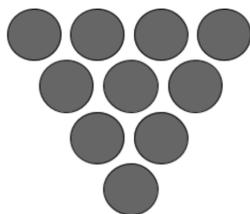
6 A Birthday Problem

Four teachers were born in different consecutive months. What is each teacher's birth month?

- Mr. Lomas was born before Mr. Kelly.
- Mr. Kelly was born two months after Mr. Melby.
- Mr. Melby was born after Mr. Cheng but before Mr. Lomas.
- Mr. Cheng was born in January.

7 Moving Dots

Rearrange the dots above into a triangle that points upward in the least number of moves possible. A move is defined as changing the location of a dot.



8 Hex Game

This is a game you play with your friend! Every turn, each person can claim a hex tile as their own by marking it! To win, you want to link a chain of hex tiles from one side to the other side. Try to find a strategy to win the game.