

1 Introduction

2 Coin Games

1. Flip a fair coin twice. What is the probability that you get two heads (HH)? $1/4$

What is the probability that you get heads followed by tails (HT)? $1/4$

Are these probabilities the same? *Yes*

2. *FYI: This is a very advanced question (contact ag7570@pleasantonusd.net for a full solution)*
Flip a fair coin repeatedly until you get two heads in a row (HH). On average, how many flips should this take? *6*

What if we flip until we get heads followed by tails (HT)? *4*

Are the answers the same? *No*

3. Let's play a game! Pick a partner, and decide whether you are HH (heads-heads) or HT (heads-tails). Now flip a coin repeatedly until you get either HH or HT. The person whose sequence is flipped first wins.

Who is more likely to win, and by how much? *Same probability of winning.*

4. Now, Between you and your partner, decide whether you are HHT or THH. Now flip a fair coin repeatedly until you guys get HHT in a row or THH. Who is more likely to win, and by how much? *Same probability of winning.*

3 Penny's Game

Let's keep playing! Pick a partner, and decide whether you are player A or B. Each of you selects a sequence of heads and tails of length 3, and shows this sequence to the other player. Now flip a penny in sets of 3. The winner is the one whose sequence is flipped first! Record the information you gather in the table below.

Mainly intuitive thinking, but actual solution is advanced (contact ag7570@pleasantonusd.net for full solution). Now, calculate the probability of your sequence. Compare it to your partner's probability. Who was more likely to win? Who actually won? How do you maximize your chance at winning this game? Is the first or second person more likely to win? *Maximize chances by making your second two tosses the same as the first two tosses in your opponent's sequence; second person*

4 Counting

How many odd numbers are there from 1 to 50? *25*

How many two digit numbers are NOT multiples of 7? *77*

5 Counting AND Probability (Challenge)

1. How many non-multiples of 7 from 1-100 are odd? *43*
2. Suppose we order a surprise pizza with two toppings. The pizza toppings can be pepperoni, sausage, green peppers, onions, mushrooms, and pineapple. What is the probability that we get a pizza with onions and pineapple? *1/15*

6 Coin Prediction

Tired of coins yet? I hope not, because this last game is about coins too. I have 100 coins. Half of them are face up, half of them are face down. I will keep showing each coin one by one. If you say stop, and the next coin is a head, I win. If the next coin is a tail, you win. If you pick the second to last coin, the last coin will decide the winner or loser. What should you do to win?

Say "stop" once you are shown the 50th head because chances are that there are 50 heads and 50 tails, so once you exhaust the 50 heads, the remainder should be tails.